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# Observables and Transformation Properties of Fermions and Parafermions Constructed in Terms of Bosons. I. The Physical Variables

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In a previous paper (Kálnay, MacCotrina & Kademova, 1973), hereafter referred to as A, it was shown that the quantum Fermi or para-Fermi commutation rules of fields $f$, could be reproduced in terms of functions of Bose fields $b, b^+$,

$$f(b, b^+) \qquad \text{and} \qquad f^+(b, b^+) \tag{1}$$

when they act on suitable Bose subspaces; called here $\mathscr{B}'$; then $f$ and $b$ are annihilation fields, $f^+$ and $b^+$ creation fields. The subspace $\mathscr{B}'$ is the representative of the Fermi or para-Fermi space $\mathscr{F}$. This work was reformulated to a simpler form by Kálnay (1974a) in a paper hereafter called B, closely related to those by Kálnay & MacCotrina (1974) which we call C, and by Kálnay (1974b), hereafter called D. Refer to these papers for further literature on the subject and for the physical discussions. We shall use without further explanation the notation used in B and C. Then, if $\mathscr{B}' = \mathscr{B}_1$, $f$ is a Fermi field, if $\mathscr{B}' = \mathscr{B}_p$, $f$ is a para-Fermi field of order $p$ of parastatistics.

The present research concerns the physical variables. Let $\Omega$ be a physical variable such as energy. We call $\Omega^{\mathscr{B}}(b, b^+)$ that of the Bose field $b_\xi(\mathbf{x})$ and

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$\Omega_{\mathscr{F}}(f, f^+)$ that of the Fermi or para-Fermi system in the standard field theory, in which they are primary entities, not Bose constructed. We refer to

$$\Omega_{\mathscr{B}}(b, b^+) = \Omega_{\mathscr{F}}[f(b, b^+), f^+(b, b^+)] \tag{2}$$

Since A and B, (1) preserves all the algebraic and (when acting on $\mathscr{B}'$, Bose representative of $\mathscr{F}$) operator properties of $f$ and $f^+$, it results that $\Omega_{\mathscr{B}}$ has the same spectra and structure of eigenvectors as the original $\Omega_{\mathscr{F}}$. Thus, we found the following:

*Theorem.* The rule for obtaining the representatives of all Fermi and para-Fermi variables such that their explicit expression in terms of $f$ and $f^+$ is known in the standard theory, is given by equation (2).

The Fermi or para-Fermi system is constructed in Bose terms with the aid of a quantum entity (the Bose field) and a $c$-number entity. The $c$-number can be interpreted as a classical field and (cf. papers B and D), as such, it can be a partial carrier of physical variables such as energy or angular momentum. (Thus, generally $\Omega_{\mathscr{B}} \neq \Omega^{\mathscr{B}}$.) However, the Fermi or para-Fermi physical variable $\Omega_{\mathscr{B}}$ can be transformed, as seen in C, to a form which is consistent with that of usual Bose variables.

In the case when $\Omega$ is the Hamiltonian $H$, it results from the above that the Bose system must evolve in time according to $H^{\mathscr{B}}$ while the Fermi or para-Fermi system evolves according to $H_{\mathscr{B}}$. In the Appendix of A we constructed a $H_{\mathscr{B}}$ such that the equation of motion of $f$ be that of Dirac's electron. By comparison we see that the rule (2) applies for that case.

Then we have the Bose construction

$$f_\xi(z) = \exp(iH_{\mathscr{B}} z_0) f_\xi(\mathbf{z}) \exp(-iH_{\mathscr{B}} z_0) \tag{3}$$

of a time-dependent quantum Fermi field. Here $z = (z^0, \mathbf{z})$. In (3) it is understood that $f(\mathbf{z})$ and $H_{\mathscr{B}}$ are Bose represented according to (1) and (2), $f(\mathbf{z})$ stands for $f(b, b^+)|_{\mathbf{z}}$. When the abbreviated form (1) is made explicit as in equation (1.5a) of B, then the $\mathbf{z}$ dependence becomes visible. In contrast, with $x = (x^0, \mathbf{x})$ we have

$$b_\zeta(x) = \exp(iH^{\mathscr{B}} x_0) b_\zeta(\mathbf{x}) \exp(-iH^{\mathscr{B}} x_0) \tag{4}$$

We remark that no Fock representation of the Fermi or para-Fermi fields is assumed in this paper (cf. B). If the Bose construction given by equations (2.1.6) and (2.2.4) of A would be used for the Fermi field, the resulting correspondence would be to a special representation of the $F_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}')$ related to the $t = 0$ instant. Therefore it should be proven that equation (3) does not privilege the $t = 0$ instant. However, we now have the generalised equation (1.5) of B which does not require any special representation of the $F_{\xi\zeta\zeta'}(\mathbf{z}, \mathbf{x}, \mathbf{x}')$. A change of the origin of times only induces, as regards the Bose representation of fermions and para-fermions, a unitary transformation between equal-footing representations.

The results of this paper are: (i) The above theorem for the Bose representation of physical variables, and (ii) equation (3). This last equation is relevant because the Bose representation of fermions made explicit in papers A and B were those of time-independent fields.

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